1.6. ▶ BOATS AND STREAMS

In class X we have studied speed, distance and time which are related as

Distance = Speed × Time

In this section we shall discuss numerical problems related to boats and stream when they are rowing/sailing along the direction of stream or opposite to it by using the concept of speed, distance and time.

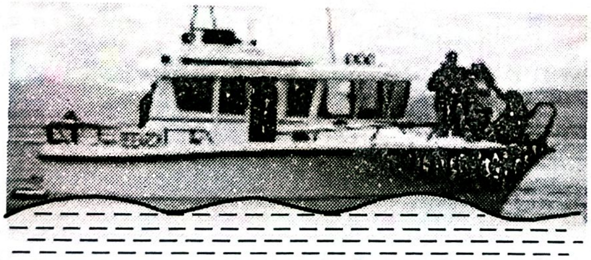
Some important terms related to boats and streams :

(*i*) **Downstream.** If the boat rows/sails along the direction of stream, then it is said to move downstream and the speed at which the boat is moving is called *downstream speed*.

(*ii*) **Upstream :** If the boat rows/sails in the direction opposite to that of the stream, then it is said to move upstream and the speed at which the boat is moving is called *upstream speed*.

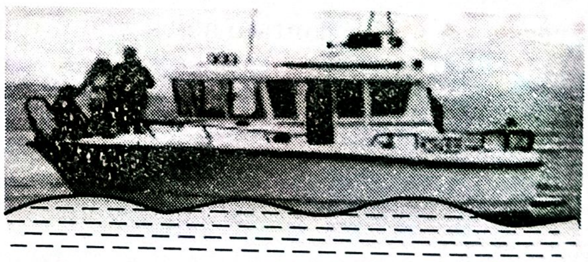
(*iii*) **Still water :** Under this circumstance, the water is considered to be stationary and the speed of stream is zero.

Downstream



Flow of the stream

Upstream



Flow of the stream

1.6.1. Some Important Results :

1. Let u be the downstream speed and v be the upstream speed of the boat.

   If the speed of boat in still water is x km/hr and speed of stream is y km/hr, then

   Downstream speed of boat $= u = (x + y)$ km/hr ...(1)

   Upstream speed of boat $= v = (x - y)$ km/hr ...(2)

$\therefore$ From (1) and (2), $\quad u + v = 2x \Rightarrow x = \dfrac{u+v}{2}$

and $\quad u - v = 2y \Rightarrow y = \dfrac{u-v}{2}$

Hence, the speed of the boat in still water $= \dfrac{u+v}{2}$

and the speed of stream (current) $= \dfrac{u-v}{2}$.

2. If a boat covers a distance of d km downstream in t_1 hours and covers the same distance d km upstream in t_2 hours, then

$$t_1 = \frac{d}{x+y} \text{ and } t_2 = \frac{d}{x-y} \qquad \left[\because \textit{Time} = \frac{\textit{distance}}{\textit{speed}}\right]$$

$$\therefore \quad \frac{t_1}{t_2} = \frac{\dfrac{d}{x+y}}{\dfrac{d}{x-y}} = \frac{x-y}{x+y} \qquad \text{...(3)}$$

Thus, while covering the same distance downstream and upstream,

$$\frac{\text{Time taken in downstream}}{\text{Time taken in upstream}} = \frac{x-y}{x+y}.$$

3. If the speed of boat in still water is x km/hr and speed of stream is y km/hr and t_1, t_2 are the times taken to cover the same distance d downstream and upstream respectively, then

$$\frac{t_1}{t_2} = \frac{x-y}{x+y} \qquad [\textit{Using (3)}]$$

$$\Rightarrow \quad \frac{t_2}{t_1} = \frac{x+y}{x-y}$$

Applying componendo and dividendo, we have

$$\frac{t_2+t_1}{t_2-t_1} = \frac{x+y+x-y}{x+y-x+y} = \frac{2x}{2y} \Rightarrow \frac{t_2+t_1}{t_2-t_1} = \frac{x}{y}$$

$$\therefore \quad x = y\left(\frac{t_2+t_1}{t_2-t_1}\right).$$

4. If the speed of boat in still water is x km/hr and speed of stream is y km/hr and the boat goes d km downstream in t_1 hour and covers the same distance d km upstream in t_2 hours, then

$$t_1 = \frac{d}{x+y} \text{ and } t_2 = \frac{d}{x-y} \qquad \text{...(4)}$$

Total distance covered $= d + d = 2d$

Total time taken $= (t_1 + t_2)$ hours

$$\therefore \qquad \text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{2d}{t_1 + t_2} = \frac{2d}{\dfrac{d}{x+y} + \dfrac{d}{x-y}} \qquad \textit{[Using (4)]}$$

$$= \frac{(x+y)(x-y)}{x}$$

Hence, **average speed of boat** $=$ $\dfrac{\textbf{downstream speed} \times \textbf{upstream speed}}{\textbf{speed in still water}}$.

SOLVED EXAMPLES

EXAMPLE 1. *A boat rows upstream at 20 km/hr and downstream at 28 km/hr. Find the speed of boat in still water and the speed of stream.*

Solution. Let the speed of boat in still water be x km/hr and speed of stream be y km/hr.

$$\therefore \qquad \text{Upstream speed} = x - y = 20 \qquad \text{...(1)}$$

$$\text{Downstream speed} = x + y = 28 \qquad \text{...(2)}$$

Adding (1) and (2), we get

$$2x = 48 \Rightarrow x = 24$$

Putting $x = 24$ in (1), we get

$$24 - y = 20 \Rightarrow y = 4$$

$\therefore$ Speed of boat in still water is **24 km/hr** and speed of stream is **4 km/hr.**

Aliter :

Downstream speed (u) = 28 km/hr

Upstream speed (v) = 20 km/hr

$$\left.\begin{aligned} \therefore \text{ Speed of boat in still water } (x) &= \frac{u+v}{2} = \frac{28+20}{2} = \textbf{24 km/hr} \\ \text{Speed of stream } (y) &= \frac{u-v}{2} = \frac{28-20}{2} = \textbf{4 km/hr.} \end{aligned}\right\} \textit{[See result 1.6.1(1)]}$$

EXAMPLE 2. *A sailor goes 8 km downstream in 40 minutes and returns in one hour. Determine the speed of the sailor in still water and speed of the current.*

Solution. Let the speed of the sailor in still water be x km/hour and the speed of the current be y km/hour.

$$\therefore \qquad \text{Upstream speed} = (x - y) \text{ km/hr}$$

$$\text{Downstream speed} = (x + y) \text{ km/hr}$$

The sailor goes 8 km downstream in 40 minutes *i.e.*, $\dfrac{40}{60}$ hours

$$\therefore \qquad \frac{8}{x+y} = \frac{40}{60} \qquad \left[\because \frac{\textit{Distance}}{\textit{Speed}} = \textit{Time}\right]$$

$\Rightarrow \qquad \dfrac{8}{x+y} = \dfrac{2}{3} \quad \Rightarrow \quad \dfrac{4}{x+y} = \dfrac{1}{3}$

$\Rightarrow \qquad x + y = 12$ ...(1)

Also, the sailor goes 8 km upstream in 1 hour.

$\therefore \qquad \dfrac{8}{x-y} = 1 \quad \Rightarrow \quad x - y = 8$ ...(2)

Adding (1) and (2), we get $\quad 2x = 20 \quad \Rightarrow \quad x = 10$

Putting $x = 10$ in (1), we get $\quad 10 + y = 12 \quad \Rightarrow \quad y = 2$

Hence, the speed of sailor in still water is **10 km/ hr** and speed of current is **2 km/ hr.**

Aliter :

$$\text{Downstream speed } (u) = \frac{8}{\frac{40}{60}} = 12 \text{ km/hr}$$

$$\text{Upstream speed } (v) = \frac{8}{1} = 8 \text{ km/hr}$$

$$\therefore \quad \text{Speed of boat in still water} = \frac{u+v}{2} = \frac{12+8}{2} = \mathbf{10\ km/hr}$$

$$\text{Speed of current} = \frac{u-v}{2} = \frac{12-8}{2} = \mathbf{2\ km/hr.}$$

EXAMPLE 3. *The speed of a boat in still water is 15 km / hr. It takes thrice as long to go upstream to a point as to return downstream. What is the speed of stream ?*

Solution. Let the distance covered (single side) be d km and y be the speed of the stream. Here, the speed of boat in still water is 15 km/hr

$\therefore \qquad$ Upstream speed $= (15 - y)$ km / hr

$\qquad$ Downstream speed $= (15 + y)$ km / hr

$\therefore \qquad$ Time taken by boat upstream $= \dfrac{d}{15-y}$ $\qquad \left[\because \textit{Time} = \dfrac{\textit{Distance}}{\textit{Speed}}\right]$

$\qquad$ Time taken by boat downstream $= \dfrac{d}{15+y}$

According to the given condition,

$$\frac{d}{15-y} = 3\left(\frac{d}{15+y}\right) \quad \Rightarrow \quad \frac{15+y}{15-y} = 3$$

$$\Rightarrow \qquad 15 + y = 45 - 3y \quad \Rightarrow \quad 4y = 30 \quad \Rightarrow \quad y = \frac{15}{2}$$

Hence, the speed of stream is **7·5 km/hr.**

EXAMPLE 4. *A boat takes 4 hours to cover a fixed distance going with the current (downstream) and takes 6 hours to cover the same distance opposite the current (upstream). If the speed of current is 2 km / hr, find the speed of boat in still water.*

Solution. Let the fixed distance covered each side be d km and the speed of boat in still water be x km/hr.

Here, the speed of current is 2 km/hr.

$\therefore$ Downstream speed = $(x + 2)$ km/hr

Upstream speed = $(x - 2)$ km/hr

Now, the boat takes 4 hours to cover a distance of d km downstream and 6 hours to cover a distance of d km upstream

$$\therefore \quad \frac{d}{x+2} = 4 \qquad \text{...(1)}$$

and
$$\frac{d}{x-2} = 6 \qquad \text{...(2)}$$

Dividing (2) by (1), we get

$$\frac{\frac{d}{x-2}}{\frac{d}{x+2}} = \frac{6}{4} \Rightarrow \frac{x+2}{x-2} = \frac{3}{2}$$

$$\Rightarrow \quad 2(x+2) = 3(x-2) \Rightarrow 2x + 4 = 3x - 6 \Rightarrow x = 10$$

Hence, the speed of boat in still water is **10 km/hr.**

Aliter :

Let x km/hr be the speed of boat in still water and y km/hr be the speed of stream.

Let t_1 and t_2 be the times taken to cover the same distance d downstream and upstream respectively.

Here $t_1 = 4$ hr, $t_2 = 6$ hr and $y = 2$ km/hr

We know that $x = y\left(\dfrac{t_2 + t_1}{t_2 - t_1}\right)$ *[See result 1.6.1(3)]*

$$\Rightarrow \quad x = 2\left(\frac{6+4}{6-4}\right) = 10$$

Hence, the speed of boat in still water is **10 km/hr.**

EXAMPLE 5. *A man can row a boat at a speed of 4 km / hr in still water. If the speed of current is 2 km / hr and it takes the man 8 hours to row to a place and come back, then find the distance between two places.*

Solution. Let d km be the distance between two places.

Here, the speed of the current is 2 km/hr and speed at which a man can row a boat is 4 km/hr.

$\therefore$ Speed of boat downstream $= 4 + 2 = 6$ km /hr

Speed of boat upstream $= 4 - 2 = 2$ km/hr

$\therefore$ Time taken by man to cover d km downstream $= \dfrac{d}{6}$ hrs

Time taken by man to cover d km upstream = $\frac{d}{2}$ hrs

According to the given condition, total time taken is 8 hours.

$$\Rightarrow \quad \frac{d}{6}+\frac{d}{2}=8 \quad \Rightarrow \quad \frac{4d}{6}=8 \quad \Rightarrow \quad d=\frac{6\times 8}{4}=12 \text{ km}.$$

Hence, the distance between two places is **12 km.**

EXAMPLE 6. *A motorboat whose speed in still water is 8 km / hr starts from a point and returns back to the same point. It took 20 minutes more to cover the distance upstream than downstream. If the speed of the current is 2 km / hr, find the distance between two points.*

Solution. Let d km be the distance between two points.

Here, the speed of motorboat in still water is 8 km/hr and speed of current is 2 km/hr.

Speed of motorboat downstream = 8 + 2 = 10 km/hr

Speed of motorboat upstream = 8 – 2 = 6 km/hr

$\therefore$ Time taken by motorboat to cover d km downstream = $\frac{d}{10}$ hrs

Time taken by motorboat to cover d km upstream = $\frac{d}{6}$ hrs

According to the given condition, we have

$$\frac{d}{6}-\frac{d}{10}=\frac{20}{60}$$

$$\Rightarrow \quad \frac{4d}{60}=\frac{20}{60} \quad \Rightarrow \quad 4d=20 \quad \Rightarrow \quad d=5 \text{ km}$$

Hence, the distance between the two points is **5 km.**

EXAMPLE 7. *A motor boat whose speed is 18 km / hr in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.*

Solution. Let the speed of the stream be y km/hr.

Here, the speed of boat in still water is given to be 18 km/hr.

$\therefore$ Downstream speed = $(18+y)$ km/hr

and Upstream speed = $(18-y)$ km/hr.

Time taken by the boat to cover 24 km downstream = $\frac{24}{(18+y)}$ hour

Time taken by the boat to cover 24 km upstream = $\frac{24}{(18-y)}$ hour

According to the given condition, we have

$$\frac{24}{18-y}-\frac{24}{18+y}=1$$

Multiplying both sides by $(18+y)(18-y)$, we get

$$24(18+y)-24(18-y)=(18+y)(18-y)$$

$$\Rightarrow \quad 24(18+y-18+y)=(18)^2-(y)^2 \quad \Rightarrow \quad 48y=324-y^2$$

$$\Rightarrow \quad y^2+48y-324=0 \quad \Rightarrow \quad (y+54)(y-6)=0$$

$$\Rightarrow \quad \text{either } (y+54)=0 \text{ or } (y-6)=0 \quad \Rightarrow \quad y=-54 \text{ or } y=6$$

$$\therefore \quad y=6 \qquad \text{[Rejecting } y=-54\text{, as speed cannot be negative]}$$

Hence, the speed of the stream is **6 km/hr**.

EXAMPLE 8. *The current of a stream runs at the rate of 2 km/hr. A motor boat goes 10 km upstream and back again to the starting point in 55 minutes. Find the speed of the motor boat in still water.*

Solution. Let the speed of the motor boat in still water be x km/hr.

Since current of a stream runs at the rate of 2 km/hr, therefore

Upstream speed of boat $=(x-2)$ km/hr.

Downstream speed of boat $=(x+2)$ km/hr.

Time taken by the motor boat to cover 10 km upstream $=\dfrac{10}{x-2}$ hr.

Time taken by the motor boat to cover 10 km downstream $=\dfrac{10}{x+2}$ hr.

According to the given condition, we have

$$\therefore \quad \frac{10}{x-2}+\frac{10}{x+2}=\frac{55}{60}$$

$$\Rightarrow \quad 600(x+2)+600(x-2)=55(x^2-4)$$

[*Multiplying throughout by* $60(x+2)(x-2)$]

$$\Rightarrow \quad 600x+1200+600x-1200=55x^2-220$$

$$\Rightarrow \quad 55x^2-1200x-220=0$$

$$\Rightarrow \quad 11x^2-240x-44=0$$

$$\Rightarrow \quad (11x+2)(x-22)=0 \quad \Rightarrow x=\frac{-2}{11}, 22$$

$$\therefore \quad x=22 \qquad \left[\text{Rejecting } x=-\frac{2}{11}\text{, as speed cannot be } -ve\right]$$

Hence, the speed of the motor boat in still water is **22 km/hour.**

EXAMPLE 9. *Find the average speed of a boat in a round trip between two points A and B which are 50 km apart, if the speed of boat in still water is 25 km/hr and speed of stream is 5 km/hr.*

Solution. Here, the speed of boat in still water is 25 km/hr and speed of stream is 5 km/hr

$\therefore$ Upstream speed $=(25-5)$ km/hr

Downstream speed $=(25+5)$ km/hr

The distance between two points A and B = 50 km [(*Given*)]

Time taken in upstream journey $= \dfrac{50}{25-5} = \dfrac{50}{20} = \dfrac{5}{2}$ hours

Time taken in downstream journey $= \dfrac{50}{25+5} = \dfrac{50}{30} = \dfrac{5}{3}$ hours

$$\therefore \quad \text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{50+50}{\frac{5}{2}+\frac{5}{3}} = \frac{100}{\frac{15+10}{6}} = \frac{100}{25} \times 6 = 24 \text{ km/hr}$$

Hence, the average speed of boat is **24 km/hr.**

Aliter :

Upstream speed = (25 – 5) km/hr

Downstream speed = (25 + 5) km/hr

$$\therefore \quad \text{Average speed of boat} = \frac{\text{downstream speed} \times \text{upstream speed}}{\text{speed in still water}} \quad [\textit{See result 1.3.1(4)}]$$

$$= \frac{(25+5)(25-5)}{25} = \frac{30 \times 20}{25} = \mathbf{24 \text{ km/hr.}}$$

EXAMPLE 10. *A man goes 12 km downstream and comes back to starting point by swimming non-stop in 3 hours. If the speed of stream is 3 km/hr, find the speed with which the man can swim in still water.*

Solution. Let x km/hr be the speed of man in still water and y km/hr be the speed of stream. Here $y = 3$ km/hr.

$\therefore$ Downstream speed $= x + y = (x + 3)$ km/hr

Upstream speed $= x - y = (x - 3)$ km/hr

Total distance covered by man = 12 + 12 = 24 km

Total time taken by man = 3 hours

$$\therefore \text{ Average speed of man} = \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{24}{3} = 8 \text{ km/hr} \qquad ...(1)$$

$$\text{We know that average speed} = \frac{\text{downstream speed} \times \text{upstream speed}}{\text{speed in still water}}$$

$$\Rightarrow \quad \text{Average speed} = \frac{(x+3)(x-3)}{x}$$

$$\Rightarrow \quad 8 = \frac{(x+3)(x-3)}{x} \qquad [\textit{Using (1)}]$$

$$\Rightarrow \quad 8 = \frac{x^2-9}{x} \quad \Rightarrow \quad x^2 - 9 = 8x$$

$$\Rightarrow \quad x^2 - 8x - 9 = 0 \Rightarrow (x-9)(x+1) = 0 \Rightarrow x = 9 \text{ or } x = -1$$

$$\Rightarrow \quad x = 9 \qquad [\textit{Rejecting } x = -1 \textit{ as speed cannot be } - \textit{ve}]$$

Hence, the man can swim in still water at a speed of **9 km/hr.**

EXERCISE 1.3

1. A boat rows with a speed of 10 km/hr in still water. If the speed of stream is 3 km/hr, then find the time taken by boat to cover the distance of 26 km downstream.
2. A boat can row upstream at 10 km/hr and downstream at 18 km/hr. Find the speed of boat in still water and the speed of the current.
3. The speed of a boat in still water is 11 km/hour. It can go 12 km upstream and return downstream to the original point in 2 hours 45 minutes. Find the speed of the stream.
4. A boatman takes half as much time in rowing his boat for a certain distance downstream than upstream. What is the ratio between his speed of rowing the boat in still water and speed of current? *[CBSE Sample Paper 2023]*
5. A boatman takes twice as long as to go upstream to a point as to return downstream to the starting point. If the speed of a boat in still water is 15 km/hr, what is the speed of the stream ?
6. A man rows downstream 30 km and upstream 20 km. If he takes 5 hours to cover each distance, then find the speed at which he rows in still water and the speed of the current.
7. A boat running downstream covers a distance of 16 km in 2 hours while for covering the same distance upstream it takes 4 hours. What is the speed of boat in still water ?
8. A motor boat whose speed is 15 km/hr in still water goes 30 km downstream and comes back in a total of 4 hours 30 minutes. Find the speed of stream.
9. The speed of a boat in still water is 8 km/hr. It can go 30 km upstream and 44 km downstream in 10 hours. Find the speed of the stream.
10. The speed of a boat in still water is 15 km/hr. It can go 30 km upstream and return downstream to the original point in 4 hours 30 minutes. Find the speed of the stream.
11. A man can row three-quarters of a kilometre upstream in $11\frac{1}{4}$ minutes and downstream in $7\frac{1}{2}$ minutes. Find the speed (in km/hr) of the man in still water.
12. Speed of a boat in standing water is 9 km/hr and the speed of stream is 1·5 km/hr. A man rows to a place at a distance of 105 km and comes back to the starting point. Find the total time taken by the man.
13. Mansi can row a boat 8 km downstream and return in 1 hour 40 minutes. If the speed of the stream is 2 km/hr, find the speed of the boat in still water.
14. Sanchi can row her boat at a speed of 5 km/hr in still water. If it takes her 1 hour more to row the boat 5·25 km upstream than to return downstream, find the speed of the stream.
15. A man can row a boat in still water at the rate of 6 km/hr. If the water in a river flows at the rate of 2 km/hr, it takes 3 hours more in upstream than in downstream for the same distance. How far is the place ?
16. A boat rows a distance of 12 km downstream at 13 km/hr and covers the same distance upstream at 7 km/hr. Find the average speed of the boat.

ANSWERS

1. 2 hours	2. 14 km/hr; 4 km/hr	3. 5 km/hr	4. 3 : 1
5. 5 km/hr	6. 5 km/hr ; 1 km/hr	7. 6 km/hr	8. 5 km/hr
9. 3 km/hr	10. 5 km/hr	11. 5 km/hr	12. 24 hours
13. 10 km/hr	14. 2 km/hr	15. 24 km	16. 9.1 km/hr